No. of Printed Pages : **3**

Sl. No.

B0-R4 : BASIC MATHEMATICS

NOTE :

1. Answer question 1 and any FOUR questions from 2 to 7.

2. Parts of the same question should be answered together and in the same sequence.

Time : 3 Hours **Total Marks : 100**

1. (a) Find the polar form of the complex number $\frac{1+2i}{1-3i}$.

(b) Find the rank of the matrix :

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

(c) If the length of the diagonal of a square is increasing at the rate of 0.2 cm/sec, then find the rate of increase of its area when the length of its side is $30/\sqrt{2}$ cm.

(d) Evaluate $\lim_{x \to \pi/2} \frac{\cot x - \cos x}{(\pi - 2x)^3}$.

(e) Find the slope of tangent on the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.

(f) Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = y^2$.

(g) Check the convergence of the alternating series :

$\frac{1}{1.3} - \frac{2}{3.5} + \frac{3}{5.7} - \frac{4}{7.9} + \ldots\infty$ **(7x4)**

2. (a) If $s = a + b + c$, show that $\begin{vmatrix} s+c & a & b \\ c & s+a & b \\ c & a & s+b \end{vmatrix} = 2s^3$. **(6)**

(b) Solve the system of simultaneous linear equations using "**Cramer's rule**" : **(6)**

$2x - 3y + 5z = 16$

$3x + y - 6z = -11$

$-x + 2y + 4z = 12$

(c) Find the Eigen values and Eigen vectors of the matrix : **(6)**

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

3. (a) Find all the asymptotes of the family of curves $y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 = 0$. **(8)**

(b) Find the local maximum and local minimum of the function : **(6)**

$$f(x) = 2x^3 - 21x^2 + 36x - 30$$

(c) Find the value of c in Mean Value Theorem when $f(x) = 2x^3 - 5x^2 - 4x + 3$, $x \in [1/2, 3]$. **(4)**

4. (a) Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$. **(5)**

(b) Find the area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and x-axis. **(5)**

(c) Solve the homogeneous linear differential equation : **(8)**

$$x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 20y = (x+1)^2$$

5. (a) Check the convergence of the series : **(6)**

$$\sum_{n=1}^{\infty} \frac{2n^3 + 5}{4n^5 + 1}$$

(b) Discuss the convergence of the series : **(6)**

$$\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \ldots\infty$$

(c) Using Taylor's series, find the expansion of tan$x$ about a point $x = \pi/4$. **(6)**

6. (a) Solve the equation $x^7 + x^4 + x^3 + 1 = 0$. **(6)**

(b) Sketch a graph of the conic $r = \frac{6}{2 - \sin\theta}$ and write its Cartesian equation. **(6)**

(c) Find the vertex, focus, axis, length of latus-rectum and equation of directrix of the conic $y^2 - x - 2y + 2 = 0$. Also, trace the graph. **(6)**

7. (a) Find all the vectors of the magnitude 8 that are perpendicular to the plane of $\hat{i}+2\hat{j}+\hat{k}$ and $-\hat{i}+3\hat{j}+4\hat{k}$. **(4)**

(b) Find the vector equation of the plane passing through the points $\hat{i}+\hat{j}-2\hat{k}, 2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$. **(4)**

(c) Find the projection of the vector $2\hat{i}-\hat{j}+\hat{k}$ to the vector $\hat{i}+2\hat{j}+2\hat{k}$. **(4)**

(d) Verify that for given set of 3 vectors.

$\vec{a}=\hat{i}+\hat{j}-2\hat{k}$

$\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and,

$\vec{c}=\hat{i}+2\hat{j}+\hat{k}$,

$\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c}$ **(6)**

- o O o -