Sl. No.

CO-R4.B1 : ELEMENTS OF BASIC MATHEMATICAL SCIENCES

NOTE :
1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

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1. (a) Evaluate $\lim_{x\to\frac{\pi}{4}} \frac{\sin\left(x+\frac{\pi}{4}\right)-1}{\log\ \sin 2x}$.

(b) Find a vector of magnitude 5 perpendicular to the vectors $2\hat{i}+\hat{j}-3k$ and $\hat{i}-2\hat{j}+k$.

(c) A line passes through the point of intersection of the lines $x+y-1=0$ and $2x-y+3=0$ and is perpendicular to $2x-y+3=0$. Find its equation.

(d) Use Taylor's Theorem to expand $\sin x$ in ascending powers of $\left(x-\frac{\pi}{2}\right)$.

(e) Calculate $\int_0^{\frac{\pi}{4}}\left(1-x^2\right)\sin 2x\ dx$.

(f) Find the standard deviation for the following discrete distribution.

x	8	12	16	20	24
$p(x)$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{12}$

(g) Evaluate $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$ **(7x4)**

2. (a) Find the rank of the matrix $A=\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

(b) Find the solution to the system of equations by Gauss elimination method

$$x+2y+5z=10$$
$$x-y-z=-2$$
$$2x+3y-z=-11$$

(c) If $y=\sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right)$, then find $\frac{dy}{dx}$. **(6+6+6)**

3. (a) Find the eigen values and eigen vectors of $A=\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}$.

(b) Show that the maximum value of $x^{1/x}$ is $e^{1/e}$.

(c) Show that the equation of the parabola whose focus is (3, -4) and directrix is the line $x+y-2=0$ is $x^2-2xy+y^2-8x+20y+46=0$. **(8+5+5)**

4. (a) Evaluate $\int \frac{\sec^2\theta\, d\theta}{\sec^2\theta-3\tan\theta+1}$.

(b) Test the convergence of the series $\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+\ldots$ ....

(c) A and B are two independent events such that $P(A\cap B)=\frac{3}{25}$ and $P(A'\cap B)=\frac{8}{25}$ then find the value P(A). **(6+6+6)**

5. (a) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total of their output. 5,4 and 2 percent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B ?

(b) A coin tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased. **(9+9)**

6. (a) A die is tossed thrice. A success is getting ' 1 or 6' on a toss. Find the mean of the number of success.

(b) (i) State the Memory less property of the exponential distribution.

(ii) Consider a 2 - server system in which a customer is served first by server 1, then by server 2, and then departs. The service time at server 1 are exponential random variable with rates μ_i, i=1, 2. When you arrive, you find server 1 free and two customers at server 2 - customer A in service and customer B waiting in line.

(a) Find P_A, the probability that A is still in service when you move over to server 2.

(b) Find P_B, the probability that B is still in the system when you move over to 2.

[9+9(3+3+3)]

7. (a) Find the moment generating function of the Poisson distribution, and hence determine its mean and variance.

(b) The following table gives the number of accidents that took place in an industry during various days of a week. Test if the accidents are uniformly distributed over the week.

Day	Mon.	Tue.	Wed.	Thurs.	Fri.	Sat.
No. of accidents	14	18	12	11	15	14

Table value of χ^2 at 5% level for 5 d.f = 11.09 **(8+10)**

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