Sl. No.

C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

NOTE :
1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

1. (a) In a test of a printed circuit board using a random test pattern, an array of 10 bits is equally likely to be 0 or 1. Assume the bits are independent. What is the probability that exactly five bits are 1s and five bits are 0s ?

(b) 2.5% of mobile phone chargers fail during the warranty period provided they are kept dry. The failure percentage is 5.6, if they are ever wet during the warranty period. If 91% of the chargers are kept dry and 9% are wet during warranty period, what is the probability that a phone charger fails during the warranty period ?

(c) Arrivals at telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of phone calls is assumed to be distributed exponentially, with a mean of 3 minutes.What is the probability that a person arriving at the booth will have to wait ?

(d) Consider a binary channel with input symbols X = {0, 1}, output symbols Y = {0, 1} and the channel matrix.

$$\begin{bmatrix} 1/3 & 2/3 \\ 1/5 & 4/5 \end{bmatrix}$$

If $p_i = P\{X_i = i\}$ and denotes the probability that the symbol xi is selected for transmission, $q_j = P\{Y_j = j\}$ the probability that the symbol yj is received, further assume the input probabilities as : $p_0 = 6/7$ and $p_1 = 1/7$. Compute output probabilities q_0 and q_1.

(e) Use the graphical method to solve the following LP problem.

$$\begin{aligned} &\textit{Maximize } Z = 15x_1 + 10x_2 \\ &\textit{Subject to } 4x_1 + 6x_2 \le 360 \\ &\qquad 3x_1 + 0x_2 \le 180 \\ &\qquad 0x_1 + 5x_2 \le 200 \\ &\qquad x_1 \ge 0, x_2 \ge 0 \end{aligned}$$

(f) Find Laplace transform $L\{F(t)\}$, if :

$$F(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right) & t > \frac{2\pi}{3} \\ 0 & t < \frac{2\pi}{3} \end{cases}$$

(g) Generate 2 random numbers from the probability density function of random variable X given by $f(x) = \lambda e^{-\lambda x}, 0 < x < \infty, \lambda > 0$ Use the uniform random number generator given by $r_n = 35 r_{n-1} \,(\text{mod}\, 100), n = 0, 1, 2\ldots$ when $r_0 = 57$ and $\lambda = 2$. **(7x4)**

2. (a) A source memory has six characters with the following probabilities of transmission.

A	B	C	D	E	F
1/3	1/4	1/8	1/8	1/12	1/12

Devise the Shannon-Fano encoding procedure to obtain a uniquely decodable code to the above message ensemble. What is the average length, efficiency and redundancy of the code that you obtain ?

(b) A transmitter has a character consisting of five letters $(x_1, x_2, x_3, x_4, x_5)$ and the receiver has a character consisting of four letters, (y_1, y_2, y_3, y_4). The joint probability for the communication is given below :

$P(x_i, y_j)$	y_1	y_2	y_3	y_4	$P(x_i)$
x_1	0.25	0	0	0	0.25
x_2	0.10	0.30	0	0	0.40
x_3	0	0.05	0.10	0	0.15
x_4	0	0	0.05	0.10	0.15
x_5	0	0	0.05	0	0.05
$P(y_j)$	0.35	0.35	0.20	0.10	

(i) Determine the different entropies for the channel, assume that $0 \log 0 = 0$.

(ii) Determine *H (X), H (Y), H (X, Y)* and *H (Y | X)*. **(9+9)**

3. (a) Patients arrive at the doctor's office according to a Poisson process with rate $\lambda = \frac{1}{10}$ minute. The doctor will not see a patient until at least three patients are in the waiting room. (i) Find the expected waiting time until the first patient is admitted to see the doctor. (ii) What is the probability that nobody is admitted to see the doctor in the first hour ?

(b) Let $Y_1, Y_2, \ldots\ldots, Y_n$ be independent poisson random variable having parameters X_i, $(1 \leq i \leq n)$ and

$$X = \sum_{i=1}^{n} y_i$$

State the central limit theorem, and using it derive the approximation formula for $P(X \leq x)$. **(9+9)**

4. (a) Find the inverse Laplace Transform :

(i) $L^{-1}\left\{\dfrac{6s-4}{s^2-4s+20}\right\}$

(ii) Use convolution theorem to find $L^{-1}\left\{\dfrac{s}{\left(s^2+a^2\right)^2}\right\}$

(b) Find the Fourier coefficients corresponding to the function :

$$F(x)=\begin{cases}0 & -5<x<0\\ 3 & 0<x<5\end{cases}, \text{ Period } = 10$$

Write the corresponding Fourier series. **(9+9)**

5. (a) Suppose the joint pmf of a bivariate r.v. (X, Y) is given by :

$$Pxy\ (x,y) = \begin{cases}\dfrac{1}{3} & \text{for } (x,y) = (0,1),\ (1,0),\ (2,1)\\ 0 & \text{otherwise}\end{cases}$$

Answer the following :

(i) Are X and Y independent ?

(ii) Are X and Y uncorrelated ?

(b) Let $Z_1, Z_2, \ldots Z_n$, be independent identically distributed random variables with $P(Z_i=1)=p$ and $P(Z_i=-1)=q=1-p$ for all i. Let $X_n=\sum_{i=1}^{n} Z_i$ and $X_0=0$. Show that X_n is a Markov chain and find its transition probabilities. **(9+9)**

6. (a) Use the simplex method to solve the following LP problem.

$$\begin{aligned}
&\textit{Maximize} \quad Z=3x_1+5x_2+4x_3\\
&\textit{Subject to} \quad 2x_1+3x_2 \le 8\\
&\qquad 2x_1+5x_3 \le 10\\
&\qquad 3x_1+2x_2+4x_3 \le 15\\
&\qquad x_1\ge 0, x_2\ge 0, x_3\ge 0
\end{aligned}$$

(b) Solve the following all integer programming problem using the branch and bound method.

$$\begin{aligned}
&\textit{Maximize} \quad Z=2x_1+3x_2\\
&\textit{Subject to} \quad 6x_1+5x_2\le 25\\
&\qquad x_1+3x_2\le 10\\
&\qquad x_1\ge 0, x_2\ge 0 \textit{ and integers}
\end{aligned}$$

(9+9)

7. (a) Use Dynamic Programming algorithm and determine the value of u_1, u_2 and u_3, so as to :

$$\textit{Maximize} \quad Z = u_1 u_2 u_3$$
$$\textit{Subject to} \quad u_1 + u_2 + u_3 = 10$$
$$u_1 \geq 0, u_2 \geq 0, u_3 \geq 0$$

(b) Use Kuhn-Tucker conditions and Solve :

$$\textit{Maximize} \quad Z = 10x_1 - x_1^2 + 10x_2 - x_2^2$$
$$\textit{Subject to} \quad x_1 + x_2 \leq 9$$
$$x_1 - x_2 \geq 6$$
$$x_1 \geq 0, x_2 \geq 0$$

(9+9)

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