No. of Printed Pages : **4**

Sl. No.

C4-R4 : ADVANCED ALGORITHMS

NOTE :

1. Answer question 1 and any FOUR questions from 2 to 7.

2. Parts of the same question should be answered together and in the same sequence.

Time : 3 Hours **Total Marks : 100**

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1. (a) Solve the recurrence, T(n) = T(n/2) + T(2n/3) + n^2 :

(b) With an example, explain NP- completeness.

(c) Assume you are creating an array data structure that has a fixed size of n. You want to backup this array after every n insertion operations. Unfortunately, the backup operation is quite expensive (takes n time to do the backup). Insertions without a backup just take 1 time unit. How frequently can you do a backup and guarantee that the amortized cost of insertion is O(1) ?

(d) What is the maximal codeword length in a Huffman encoding of an alphabet of n symbols ?

(e) How can one use Prim's algorithm to find a spanning tree of a connected graph with no weights on its edges ? Is it a good algorithm for this problem ?

(f) Can sorting be done in linear time ? If yes, name the algorithms.

(g) Define approximation ratio. Explain what is meant by 2-approximation algorithms. **(7x4=28)**

2. (a) Suppose we're doing a sequence of operations (numbered 1, 2, 3,.) such that the i^{th} operation :

- costs 1 if i is **not** a power of 2
- costs i if i is a power of 2

For example, the following table shows the costs for each of the first few operations :

operation number : 1 2 3 4 5 6 7 8 9 ...

cost : 1 2 1 4 1 1 1 8 1 ...

Give the best upper bound you can on the amortized cost per operation.

(b) Construct a Huffman code for the following data :

Symbol	A	B	C	D	E
frequency	0.4	0.1	0.2	0.15	0.15

(i) Encode ABACABAD using the code above.

(ii) Decode 100010111001010 using the code. **(9x2=18)**

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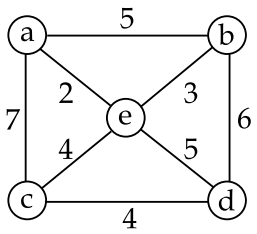
3. (a) (i) Apply Prim's algorithm to the following graph for Minimum Spanning tree.



(ii) The notion of a minimum spanning tree is applicable to a connected weighted graph.

Do we have to check a graph's connectivity before applying Prim's algorithm, or can the algorithm do it by itself ?

(iii) Does Prim's algorithm always work correctly on graphs with negative edge weights ?

(b) There are n people, each in possession of a different rumour. They want to share all the rumours with each other by sending electronic messages. Assume that a sender includes all the rumours he or she knows at the time the message is sent and that a message may only have one addressee. Design a greedy algorithm that always yields the minimum number of messages they need to send to guarantee that every one of them gets all the rumours.

(9x2=18)

4. (a) Consider two teams, A and B, playing a series of games until one of the teams wins n games. Assume that the probability of A winning a game is the same for each game and equal to p, and the probability of A losing a game is $q = 1 - p$. (Hence, there are no ties.) Let P(i, j) be the probability of A winning the series if A needs i more games to win the series and B needs j more games to win the series.

(i) Set up a recurrence relation for P(i, j) for dynamic programming algorithm.

(ii) Find the probability of A winning a 7-game series if A's probability of winning a game is 0.4.

(iii) Write pseudocode of the DP algorithm and determine its time and space efficiencies.

(b) Apply Bottom up Dynamic Programming approach to find the maximum profit earned in following example of O/I Knapsack :

Item	Weight	Profit
1	4	$6
2	2	$4
3	3	$5
4	1	$3
5	6	$9
6	4	$7

capacity W=10.

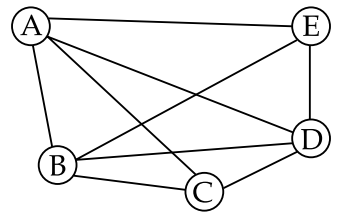
(9x2=18)

5. (a) (i) Here is an array partitioned by the first step of quicksort : 3, 0, 2, 4, 5, 8, 7, 6, 9. Which of these elements could be the pivot in the first step of quick sort ?

(ii) Here is an array of ten integers : 5 3 8 9 1 7 0 2 6 4.

Considering 5 as pivot partition this array using quicksort's partition function. Draw the resulting array after partition finishes.

(b) Use Triangle inequality to find approximate solution to the Travelling Salesman Problem for the graph given below.



(9x2=18)

6. (a) To enjoy a weekend, a group of N students of a school went to lake area for boating.

All boats in this lake can be used by maximum 3 persons at a time.

Weighing capacity of each boat is 150 kg. In this group of N students, there are many students who do not like each other, and don't want to sit in same boat.

Also, all students have their own preference to share the same boat; however they are not rigid to share the boat with given preference.

Following table lists the preferred preference of each student along with the list of disliking students and weight of each student.

Table 1

Student No/weight in Kg	List of Disliked Students	Preferred student to share the boat
1 / 45 kg	3, 4, 6	5
2/ 52 kg	3, 5	4
3 / 39 kg	1, 2, 5, 7	3
4 / 58 kg	1, 6, 7	4
5 / 61 kg	2, 3	4
6 / 56 kg	1, 4, 7	2
7 / 73 kg	3, 4, 6	1

Propose an efficient algorithm to find the minimum number of boats required for boating depending upon above constraints. Trace your proposed algorithm to find boat-wise list of students in Table 1.

(b) Considering unlimited availability of the coins of denominations 1, 2, 5 and 10. Apply the dynamic programing approach to find minimum number of coins required to change the amount 17.

(9x2=18)

7. (a) Given a set of points {p1, . . . ,pn}. Find the pair of points {pi ,pj} that are closest together in O(n log n) time.

(b) (i) Two strings, say A & B are compared for exact match using Naïve String Matching algorithm. Considering the length of the strings A and B as m and n respectively, where m > n , analyse the time complexity to compare A and B using Naïve String Matching.

(ii) Define prefix function that is used by the Knuth/Morris/Pratt algorithm.

(iii) Show the Knuth/Morris/Pratt prefix function for pattern aabaabcab.

(9x2=18)

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