B0-R4: BASIC MATHEMATICS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Express $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ in the form of $x+iy$ where *x* and *y* are real numbers.

b) Find $\frac{dy}{dx}$ when $x = \log t + \sin t, \quad y = e^t + \cos t$.

c) If one root of the equation $\begin{vmatrix} 7 & 6 & x \\ 2 & x & 2 \\ x & 3 & 7 \end{vmatrix} = 0$ is $x = -9$, then find the other roots.

d) Evaluate $\int \frac{x-1}{(x-2)(x-3)} dx$.

e) If $\vec{a} = i - 2j + k$, $\vec{b} = 2i + j + k$ and $\vec{c} = i + 2j - k$ then determine the vector $\vec{a} \times (\vec{b} \times \vec{c})$.

f) Find the equation of the hyperbola whose conjugate axis length is 5 and the distance between the foci is 13.

g) Test the convergence of series $1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} + \frac{1}{5^5} \cdots$.

(7x4)

2.

a) Determine the values of λ and μ so that the system of linear equations

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

b) Show that the characteristics equation of a matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, is $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$. Use it to find A^{-1}.

(9+9)

3.

a) If $\sin y = x \cos(a + y)$, then show that

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a} .$$

Find the value of $\frac{dy}{dx}$ at *x* = 0.

b) Discuss the applicability of the Lagrange mean value theorem for the function $f(x) = x(x-1)(x-2)$ defined in the interval $\left[0, \frac{1}{2}\right]$.

c) Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$.

(6+6+6)

4.

a) Find the area of the region S = { (x, y): $x^2 + y^2 \le 16$, $y^2 \le 4x$}

b) Find the equations of oblique asymptotes to the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.

(9+9)

5.

a) Test the convergence of the series $1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots$ for all real values of x.

b) Find the equations to the straight lines passing through the points $(3, -2)$ and inclined at 60° to the line $\sqrt{3}x + y = 1$.

c) Find the equation of circle with centre (1, -3) and it touches the line 2x – y – 4 = 0.

(8+6+4)

6.

a) Find eccentricity, coordinates of foci and length of the latus rectum of the ellipse $4x^2 + 3y^2 = 36$. Also find an equation of the tangent to the ellipse at the point $(3, -2)$.

b) Let a be a non-zero number. Examine the function $f(x, y) = xy + \frac{a^3}{x} + \frac{a^3}{y}$ for maxima and minima.

(9+9)

7.

a) Evaluate $\int_1^2 \frac{dx}{x(x+1)^2}$.

b) Let $\vec{a} = 2i + k, \vec{b} = i + j + k$ and $\vec{c} = 4i - 3j + 7k$. Determine a vector $\vec{p}$ such that $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$.

c) Evaluate $\lim_{x \to 0} \left\{ \frac{xe^x - \log(1+x)}{x^2} \right\}$.

(8+6+4)