B0-R4: BASIC MATHEMATICS

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Express the complex rational expression $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ in the form of $x+iy$, such that *x* and *y* are real numbers.

b) Evaluate $\lim_{x \to 0} x\log(\sin x)$.

c) Show that the matrix $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is an orthogonal matrix.

d) Discuss the convergence or divergence of the following series:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots\cdots\cdots\cdots\cdots$$

e) Solve the differential equation $\frac{dy}{dx}+\frac{y}{x}=y^2$.

f) Find the length of the curve $y=\log(\sec x)$ between the points $x=0$ and $x=\frac{\pi}{3}$.

g) Using the properties of the definite integrals, evaluate, $\int_0^{\pi/2} \frac{\cos x-\sin x}{1+\sin x\cos x}\,dx$.

(7x4)

2.

a) Find the inverse of a matrix $A=\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ by the Gauss Elimination method. Show the steps involved in it.

b) Let α and β be the roots of the equation $a\tan\theta+b\sec\theta=c$. Convert the given equation into a quadratic equation in $\tan\theta$, and then prove that $\tan(\alpha+\beta)=\frac{2ac}{a^2-c^2}$.

(10+8)

3.

a) Find all the points of local maxima and minima of the function $f(x,y)=x^3+y^3-3xy$.

b) Find all the asymptotes of the curve $2x(y-3)^2=3y(x-1)^2$.

(8+10)

4.

a) Find the area of the region bounded between the parabola $y^2 = 2x$ and the line $y = 4x - 1$.

b) Find the first three terms of the Taylor's series expression of $\sin x$ about $x = \frac{\pi}{2}$.

(9+9)

5.

a) Find the volume of solid generated by the revolution of the curve $y = \sin x$ from $x = 0$ to $x = 2\pi$ about x- axis.

b) Let $f(x) = \frac{x}{1 + e^{1/x}}, x \neq 0, f(0) = 0$. Prove that f is continuous but not differentiable at $x = 0$.

c) Solve the differential equation $(x+1)\frac{dy}{dx} - y = e^x (x+1)^2$.

(6+6+6)

6.

a) Find the unit vector perpendicular to the plane of two vectors $3i + 2j - k$ and $12i + 5j - 5k$. Also, determine the sine of the angle between them.

b) Find the equation of the parabola whose focus is the point (2, 3) and whose directrix is the straight line $x - 4y + 3 = 0$. Also find the length of its latus rectum.

(8+10)

7.

a) Find the characteristic roots and characteristic vectors of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

b) Find the equation of the circle which passes through the points $(1, -2)$ and $(4, -3)$ and its centre lies on the line $3x + 4y = 7$.

(10+8)