B3.2-R4: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Show that the function f:R – {-1} → R –{1} given by f(x)= $\frac{x}{x+1}$ is one-one, onto function.

b) Show that

P↔Q and ((~P)VQ)∧(P∨(~Q))

are equivalent statements (where ~ denotes negation of a statement).

c) Find the solution of the following recurrence relation

$S_n=S_{n-1}+S_{n-2}$, with $S_0=S_1=3$.

d) Determine the number of edges in a graph having 6 vertices; two vertices have degree 4 and four vertices have degree 2.

e) Evaluate the value of the Boolean expression x +(yz)' for the triplets (0,1,0) and(0,1,1), where ' is used for complement.

f) Express E(x, y, z) = x(y' z)' in its complete sum-of-products form, where ' is used for complement.

g) When a bank customer inserts his debit card into ATM, it requests him to input his secret identification number (ID). Suppose the ID is 234. Design a finite state automata that model the ID number in an ATM.

(7x4)

2.

a) In how many ways can 5 Physics books, 4 Mathematics books and 2 Computer science books be arranged on a shelf so that all books of the same subject are together?

b) Use mathematical induction to show that

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1.$$

c) A coin is tossed 5 times. What is the probability of getting at least three heads?

(6+6+6)

3.

a) Is the following argument valid?

If two sides of a triangle are equal, then the opposite angles are equal.

Two sides of a triangle are not equal.

∴the opposite angles are not equal.

b) Let a(n) and b(n) be sequences of positive numbers. If s(n) =O(a(n)) and t(n) =O(b(n)) , then prove that s(n)+t(n)=O(max{a(n),b(n)}), where 'O' is the Big O notation.

c) Suppose the relation **R** is represented by the following matrix:

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Is R reflexive, symmetric and anti-symmetric? Justify

(6+6+6)

4.

a) Find the numeric function corresponding to the generating function

$$A(z) = \frac{3z}{(1-z)(1+2z).}$$

b) Find the value of x such that

$1!+2!+3!+4!+5!+.......+100! \equiv x(\text{mod}5)$

c) Apply the Euclid's method to find the integers m and n such that 26m+120n =2. Show all steps involved it.

(6+6+6)

5.

a) Sort the following list in ascending order using the bubble sort algorithm. Describe the steps of the algorithm in detail

3 9 6 4 1 5

b) A computing machine has been given instruction which computes the sum of three numbers. How many times the addition instruction will be executed to perform the sum of 11 numbers?

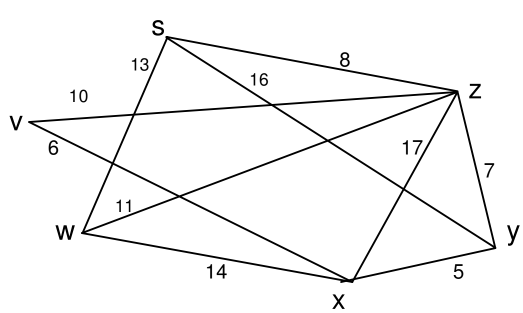
c) Let (L, $\leq$) be a lattice and a,b,c$\in$L such that $a \leq b \leq c$. Then show that

$(a \wedge b) \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$.

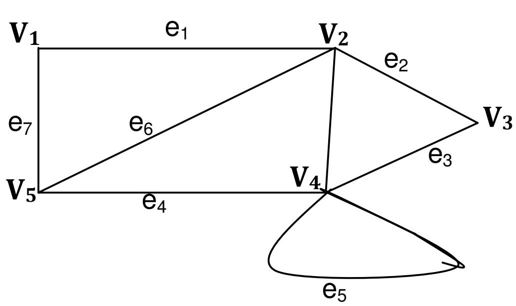
(6+6+6)

6.

a) Use the Dijkstra algorithm to find the shortest path from node s to each vertex in the graph given below:



b) Find the adjacency matrix and the incidence matrix for the following graph:

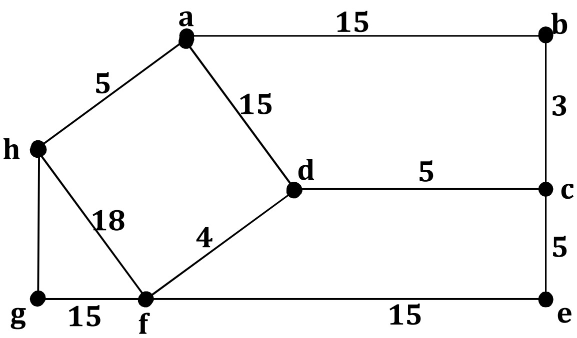


c) By using the pigeonhole principle, show that if any five numbers from 1 to 8 are chosen then two of them will add to 9.

(8+6+4)

7.

a) Consider the following railway network of cities with travelling costs between them on edges:



Find the railway network (or spanning tree) with the minimum cost.

b) Draw the Hasse diagram for the poset (P({a,b,c}),⊆) and answer the following question, where P({a,b,c) is the power set of {a,b,c}:

i) What is the maximal element of the poset?

ii) What is the minimal element of the poset?

iii) Find lub{a,c}, lub{a,b}, glb{a.c}, glb({a,b},{b,c}), where lub: least upper bound and glb: greatest lower bound

(10+8)