C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Coefficient of correlation between two variants X and Y is 0.3. The covariance is 9. The variance of X is 16. Find the standard deviation of Y series.

b) Find the Maclaurin's series of $f(x)=\sin x$.

c) Using Cramer's rule to solve the system of equations: $x+y=8;\quad x-y=4$.

d) Find the equation of the line passing through the points (1, -1) and (3, 6).

e) Find the characteristic roots of the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

f) Evaluate: $\lim_{x \to 0} \frac{e^{mx}-1}{x}$.

g) Test the convergence of series $\sum_{n=0}^{\infty}\left(\frac{n}{1+n}\right)^{n^2}$.

(7x4)

2.

a) A manufacturer knows that the condensers he makes contain on an average 1% of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensers?

b) Find an equation of the parabola with vertex at (0 , 2) and focus at (0 , 6).

c) Find the eccentricity and the length of latus rectim of the ellipse $9x^2 + 4y^2 = 36$.

(8+5+5)

3.

a) Obtain the rank of matrix $A=\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$.

b) Ten percent of screws produced in a certain factory turn out to be defective. Find the probability that in a sample of 10 screws chosen at random, exactly two will be defective.

c) A random variable x has the following probability function:

x	-2	-1	0	1	2	3
p(x)	0.1	*k*	0.2	2*k*	0.3	*k*

Find the value of *k* and calculate the mean and variance.

(6+6+6)

4.

a) Evaluate the following integrals:

i) $\int x^2 e^{2x}\ dx$

ii) $\int \frac{dx}{(x+2)(x-3)}$

b) Expand $f(x) = 3x^3 + 9x^2 + 5$ in powers of $(x-1)$.

(12+6)

5.

a) Test the convergence of the following infinite series:

i) $\sum_{n=0}^{\infty} \frac{2n^5 + 4}{n^8 - 5}$

ii) $\sum_{n=1}^{\infty} n!\, e^{-n}$

b) Verify the Lagrange's Mean value theorem for the function $f(x) = x^2$ on $[0, 9]$.

(12+6)

6.

a) Examine whether the following system of equations is consistent? If yes then solve it completely.

$x + y + z = -3$,
$3x + y - 2z = -2$,
$2x + 4y + 7z = 7$

b) A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.

(10+8)

7.

a) Find the regression lines *y* on *x* and *x* on *y* from the following data:

x	1	2	3	4	5
y	2	5	3	8	7

b) Find the equation of the circle with center (1, -3) and it touches the line $2x - y - 4 = 0$.

(10+8)