C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours   **Total Marks: 100**

1.

a) Evaluate $\lim\limits_{x \to 0} \dfrac{(1 - \cos 2x)\sin 3x}{x^2 \sin 2x}$.

b) Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfy the equation $x^2 - 3x - 7 = 0$.

c) Find the covariance between x and y

x	1	2	3	4	5
y	2	4	6	8	10

d) Evaluate $\displaystyle\int_2^4 \dfrac{x}{x^2 + 1}\,dx$.

e) Find the first two terms of the Taylor's series expression of $\sin x$ about $x = \dfrac{\pi}{2}$.

f) Find the equation of the ellipse whose foci are $(\pm 4,\ 0)$ and the eccentricity is $\dfrac{1}{3}$.

g) Two dice are thrown. Find the probability of getting an odd number on the first dice and a multiple of 3 on the other.

(7x4)

2.

a) For what real values of λ and μ, the system of linear equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

b) If x, y, z are all different real numbers and given that

$$\begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0, \text{ then prove that } 1 + xyz = 0.$$

(9+9)

3.

a) If $y = \log \tan \dfrac{x}{2} + \sin^{-1}(\cos x)$, then find $\dfrac{dy}{dx}$.

b) A box contains 400 bolts. Out of which probability of a defective bolt is 0.1. Find the mean and standard deviation for the distribution of bolts.

c) Let X be a random variable with probability distribution

$$X \quad : \quad 0 \qquad 1 \qquad 2 \qquad 3$$

$$f(X): \frac{1}{3} \qquad \frac{1}{2} \qquad 0 \qquad \frac{1}{6}$$

Find the expected value of $(X-1)^2$.

(6+6+6)

4.

a) Find the equation of a circle which passing through the point $(2, 4)$ and the centre at the intersection of the lines $x - y = 4$ and $2x + 3y = -7$.

b) Evaluate the integral $\int \dfrac{dx}{(x+1)^2 (x^2 + 1)}$.

(8+10)

5.

a) Find the maximum and minimum values of $f(x) = \sin x + \dfrac{1}{2}\cos 2x$ in $\left[0, \dfrac{\pi}{2}\right]$.

b) Test the convergence of the series

$$\frac{x}{1} + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \cdots\cdots\cdots\cdots$$

(9+9)

6.

a) Show that the equation $y^2 - 4y - 4x - 8 = 0$ represents a parabola. Find its vertex, focus and directrix.

b) Fit a straight line for regression equation of y on x

X	0	1	2	3	4	5	6
Y	2	1	3	2	4	3	3

(9+9)

7.

a) If α, β are the roots of $3x^2 - 4x + 1 = 0$, form the equation whose roots are $\dfrac{\alpha^2}{\beta}, \dfrac{\beta^2}{\alpha}$.

b) Find the limit when $n \to \infty$ of the series

$$\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots\cdots\cdots + \frac{1}{2n}.$$

(8+10)