B0-R4: BASIC MATHEMATICS

«QP_SRLNO»

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Express the complex number $\left(\frac{2+i}{3-i}\right)^2$ in to polar form.

b) Evaluate: $\lim_{x\to 0}\frac{xe^x-\log(1+x)}{x^2}$.

c) Show that $A=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$ is an orthogonal matrix.

d) Test the convergence of the series $\frac{14}{1^3}+\frac{24}{2^3}+\frac{34}{3^3}+....$

e) Solve the differential equation $x\frac{dy}{dx}-\cot y=0$.

f) Find the projection of the vector $\vec{B}=2\vec{i}+4\vec{j}-\sqrt{5}\vec{k}$ on the vector $\vec{A}=2\vec{i}-4\vec{j}+\sqrt{5}\vec{k}$. Also, find the scalar component of $\vec{B}$ in the direction of $\vec{A}$.

g) Evaluate $\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

(7x4)

2.

a) Using Gauss elimination method and back substitution solve the system of linear equations $x+y+2z=9,\quad 2x+4y-3z=1,\quad 3x+6y-5z=0$

b) Evaluate the integral $\int\frac{x+3}{x(x^2+1)}dx$.

(10+8)

3.

a) Find the area of the bounded region enclosed by the curve $y=x^3+x^2-12x$ and the x-axis.

b) Using Taylor's series about the x=3, expand $f(x)=x^4-11x^3+43x^2-60x+14$ in powers of $(x-3)$

(9+9)

4.

a) Find the maximum and minimum values of the function *f(x, y)* = $2\left(x^2-y^2\right)-x^4+y^4$.

b) Find the eccentricity of the hyperbola whose vertices are $(0,\pm 3)$ and the asymptotes are $y=\pm x$.

c) Test the convergence of the series: $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+...$.

(8+4+6)

5.

a) Verify the Lagrange mean value theorem for the function $f(x) = \log x,$ in the interval $[1, e]$.

b) Evaluate: $\lim_{x \to \frac{\pi}{2}} \dfrac{\cos 3x + 3\cos x}{\left(\dfrac{\pi}{2} - x\right)^3}$.

c) Find a unit vector perpendicular to the plane passing through the points *A*(1, -1, ,2), *B*(2, 0, -1) and *C*(0, 2, 1).

(6+6+6)

6.

a) Let a, b, c be non-zero real numbers. Determine the value of the determinant $\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix} = 0$.

b) Evaluate: $\int_1^e \dfrac{\log x}{x} dx$.

c) Find the cartesian co-ordinate of the point whose polar co-ordinates are $(\pi, \theta) = \left(7, \dfrac{2\pi}{3}\right)$.

(6+6+6)

7.

a) Find the tangent and the normal to the curve *x*(*t*) = *sec t* , *y*(*t*)=tan *t*, $-\dfrac{\pi}{2} < t < \dfrac{\pi}{2},$ at $t = \dfrac{\pi}{4}$.

b) Find the eigen value and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

(8+10)