«QP_SRLNO»

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) The probability distribution of a random variable *X* is given by $f(x) = kx^2,\ 0 \le x \le 2.$ Find (i) the value of *k* (ii) $P(1 < X < 2),$ (iii) $P(X > 1.5 / X > 1).$

b) If a random variable X takes a values 1, 2, 3, 4 such that

2P(X=1) = 3P(X=2) = P(X=3) = 5P(x=4)

Find the probability distribution of X.

c) Obtain the necessary conditions for the optimum solution of the following problem:

Minimize $f(x_1, x_2) = 3e^{2x_1+1} + 2e^{x_2+5}.$

Subject to $x_1 + x_2 - 3 = 0.$

d) Convert the following linear programming problem into the standard form:

Minimize $z = 3x_1 + 4x_2$,

Subject to

$2x_1 - x_2 - 3x_3 = -4,$

$3x_1 + 5x_2 + x_4 = 10,$

$x_1 - 4x_2 = 12,$

$x_1,\ x_3,\ x_4 \ge 0.$

e) Find a scheme for generating a random sample from the following distribution:

$$f(x) = \frac{2}{5}(x+1),\ 1 \le x \le 2.$$

f) Draw the transition diagram of the process whose one step transition matrix with three states is as follows:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0 & 0 & 1 \\ 0.25 & 0.75 & 0 \end{bmatrix} \end{array}$$

g) At a one man barber shop customers arrive in a Poisson fashion at an average rate of 4 customers per hour and the hair cutting time is exponentially distributed with an average hair cut taking 12 minutes. Find the time of waiting in the queue and probability that there are 5 customers in the system.

(7×4)

2.

a) Box A contains 1 white, 2 red, 3 green balls, Box B contains 2 white, 3 red, 1 green balls, Box C contains 3 white, 1 red, 2 green balls. Two balls are drawn from a box chosen at random and found to be one white and one red. Find the probability that the balls so drawn came from box B.

b) Find the mean recurrence time for each state of the following chain whose transition matrix is given as follows:
$$\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.4 & 0.4 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}.$$

(9+9)

3.

a) Use dual simplex method to solve the following linear programming problem:

Minimize $z = 2x_1 + x_2$,

subject to

$3x_1 + x_2 \geq 3$,

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1,\ x_2 \geq 0$.

b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & if\ |x| < 1 \\ 0, & if\ |x| > 1 \end{cases}$ and hence evaluate $\int_0^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$.

(9+9)

4.

a) Cars arrive in a pollution testing centre according to Poisson distribution at an average rate of 15 cars per hour, the testing centre can accommodate at maximum 15 cars. The service time per car is an exponential distribution with mean rate 10 per hours, find (i) the effective arrival rate at the pollution testing centre, (ii) the probability that an arriving car has not to wait for testing (iii) the probability that an arriving car will find a vacant place in the testing centre.

b) The World Series is a seven-game series that terminates as soon as either team wins four games. Let X be the random variable that represents the outcome of a World Series between teams A and B; possible values of X are AAAA, BABABAB, and BBBAAAA. Let Y be the number of games played, which ranges from 4 to 7. Assuming that A and B are equally matched and that the games are independent, calculate H(X) and H(Y).

(9+9)

5.

a) The joint PDF of (X,Y) is given by

$$f(x,y) = \begin{cases} 24xy, x > 0, y > 0, x + y \leq 1 \\ 0,\ elsewhere \end{cases}$$

Find the conditional mean and variance of Y given X.

b) A petrol station has two pumps. The service time follows exponential distribution with mean 4 minutes and cars arrive for service in a Poisson fashion at the rate of 10 cars per hour, find (i) the probability that a customer has to wait for service, and (ii) the proportion of time the pump remains idle.

(9+9)

6.

a) Three persons A, B, C are throwing a ball to each other. A always throw the ball to B and B always throws the ball to C. but C is just as likely to throw the ball to A as to B. Assuming the process to be Markovian, find the transition probability matrix and classify the states.

b) Use the branch and bound technique to solve the following integer programming problem:

Maximize $z = 7x_1 + 9x_2$,

subject to

$-x_1 + 3x_2 \leq 6$,

$7x_1 + 3x_2 \leq 35$,

$x_2 \leq 7$,

$x_1,\ x_2 \geq 0$ and are integers.

(9+9)

7.

a) Find the Laplace transform of the function $f(t) = \dfrac{1-\cos t}{t^2}$.

b) Use the KKT conditions to solve the following nonlinear programming problem:

Maximize

$$f(x_1, x_2) = x_1^2 + x_1 x_2 - 2x_2^2$$

subject to

$$4x_1 + 2x_2 \le 24,$$
$$x_1,\ x_2 \ge 0.$$

(9+9)