NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Find the convolution of two sequences $x(n) = \alpha^n u(n)$ and $h(n) = \beta^n u(n)$ when $\alpha \neq \beta$.

b) Consider a system described by the difference equation $y(n) - \frac{1}{5}y(n-1) = x(n)$ and $y(-1) = k$. Find the impulse response of the system.

c) Determine whether or not each of the following systems is shift-invariant:
i) $y(n) = x(n^2)$ ii) $y(n) = x(n)u(n)$

d) Determine the z-transform of the signal $x(n) = (\frac{1}{2})^n u(n+2) + (3)^n u(-n-1)$.

e) Compute the N point DFT of the signal $x(n) = \alpha^n, \quad 0 \le n < N$.

f) Find the value of $x(0)$ for the sequence that has a z-transform

$$X(z) = \frac{z}{\left(1 - \frac{1}{2}z^{-1}\right)(1 - \frac{1}{3}z^{-2})}, \quad |z| > 0.5$$

g) Perform the circular convolution of the following sequence:
$x_2(n) = 0.5\delta(n) + \delta(n-1) + \delta(n-2) + 6\delta(n-3)$

(7x4)

2.

a) Consider the sequence $x(n) = 4\delta(n) + 3\delta(n-1) + 2\delta(n-2) + \delta(n-3)$. Let $X(k)$ be the six-point DFT of $x(n)$.
i) Find the finite-length sequence $y(n)$ that has a six-point DFT

$$Y(k) = W_6^{4k} X(k)$$

ii) Find the finite-length sequence $q(n)$ that has a three-point DFT

$$Q(k) = X(2k), \quad k = 0,1,2.$$

b) Check whether the corresponding LTI system with system function

$$X(z) = \frac{-1 - 0.4z^{-1}}{1 - 2.8z^{-1} + 1.6z^{-2}}$$

is stable and causal, if the ROC is
(i) $|z| > 2$
(ii) $|z| < 0.8$
(iii) $0.8 < |z| < 2$

(9+9)

3.

a) Explain algorithm used in implementing least mean square (LMS) adaptive algorithm and derive the filter coefficient updating equation using LMS algorithm.

b) Design an IIR low-pass Chebyshev filter using bilinear transformation for the following specifications:
Passband: $0.75 \le |H(e^{j\omega})| \le 1 \qquad 0 \le \omega \le 0.25\pi$
Stopband: $|H(e^{j\omega})| \le 0.23 \qquad 0.63\pi \le \omega \le \pi$
and sampling frequency is 8 kHz.

(10+8)

4.

a) Design a band-pass linear phase FIR filter for the following filter specifications:

Lower cut-off frequency = 1.2 rad/sample,

Upper cut-off frequency = 2.3 rad/sample,

Obtain the filter coefficients using the window

$$w(n) = \begin{cases} 1 & 0 \le n \le 6 \\ 0 & otherwise \end{cases}$$

b) Consider the signal $x(n) = a^n u(n), \quad |a| < 1$

i) Determine the spectrum $X(\omega)$.

ii) The signal $x(n)$ is applied to a decimator that reduces the rate by a factor of 2. Determine the output spectrum.

iii) Show that the spectrum in part (ii) is simply the Fourier transform of $x(2n)$.

(10+8)

5.

a) Distinguish Harvard and SHARC digital signal processor architectures with diagrams.

b) With diagram, explain the structure of shift right barrel shifter and logarithmic shifter.

c) Explain why the discrete histogram equalization technique does not, in general, yield a flat histogram.

(6+6+6)

6.

a) Derive the expression of magnitude and phase response of symmetrical liner phase FIR filter with even value of filter length.

b) Consider a discrete LTI system described by the difference equation

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{5}{3}x(n-1)$$

Realize the system function using

i) Direct form structure I and II

ii) Cascade form

(9+9)

7.

a) Show that if a filter transfer function $H(u, v)$ is real and symmetric, then the corresponding spatial domain filter $h(x, y)$ also is real and symmetric.

b) Find the DFT of the sequence $x(n) = 4\delta(n) + 3\delta(n-1) + 2\delta(n-2) + \delta(n-3)$ using 4-point radix-2 decimation-in-frequency FFT algorithm. Show all intermediate results.

(9+9)