B4.1-R4: COMPUTER BASED STATISTICAL & NUMERICAL METHODS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**
3. **Only Non-Programmable and Non-Storage type Scientific Calculator allowed.**

Time: 3 Hours **Total Marks: 100**

1.

a) A speaks truth in 75 % and B in 80 % of the cases. In what percentages of cases are they likely to contradict each other in stating the same fact.

b) Find k such that the function f defined by

$$f(x) = \begin{cases} kx^2, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

is a p.d.f . Also determine $P(\frac{1}{3} < x \leq \frac{1}{2})$.

c) Determine the value of the constant K such that the p.d.f $f(x)$ is defined by

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

Find c.d.f$F(x)$ and hence evaluate $P[x > 0.5]$.

d) Evaluate $\sqrt{12}$ to five decimal place by Newton's iterative method.

e) The following data regarding the heights (y) and weights (x) of 100 college students are given:

$\sum x = 15000,\ \sum x^2 = 2272500, \sum y = 6800, \sum y^2 = 463025\ and\ \sum xy = 1022250$

Find the correlation coefficient between height and weight of students.

f) For two random variables, x and y with the same mean, the two regression equations are $y = ax + b$ and $x = \alpha y + \beta$ show that $\frac{b}{\beta} = \frac{1-a}{1-\alpha}$. Find also common mean.

g) A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance of the number of success.

(7x4)

2.

a) Solve the equations by LU factorization method.

$$x + y + z = 3$$
$$2x - y + 3z = 16$$
$$3x + y - z = -3$$

b) Interpolate the unique polynomial $p(x)$ of degree 2 such that

$$p(1) = 1, \quad p(3) = 27, \quad p(4) = 64$$

Using the Lagrange method of interpolation.

c) The table given below reveals the velocity $'v'$ of a body during the time$'t'$ specified. Find its acceleration at $t = 1.1$

t:	1.0	1.1	1.2	1.3	1.4
v:	43.1	47.7	52.1	56.4	60.8

(6+6+6)

3.

a) Consider the experiment of tossing two dice. Let X denote the absolute difference of the upturned faces. Find the probability mass function of X.

b) The contents of urns I, II and III are as follow:

1 white, 2 red and 3 black balls,
2 white, 3 red and 1 black balls,
3 white, 1 red and 2 black balls.

One urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from urns I, IIor III.

c) If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth (10^{th}) child exposed to the disease will be the third (3^{rd}) to catch it?

(6+6+6)

4.

a) The joint probability density function of X and Y is

$$f(x,y) = \frac{x+y}{21}, \quad x = 1,2,3; \qquad y = 1,2$$

Find marginal probability density functions of X and Y. Evaluate $P[X \leq 2]$ and $P[Y \leq 2]$. Find also the conditional distribution of Y for $X = x$.

b) State the central limit theorem. Also outline its importance.

(9+9)

5.

a) A car hire firm has two cars, which it hires day by day. The number of demands for a car on each day is distributed as a poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused ($e^{-1.5} = 0.2231$).

b) If X and Y are two random variables having joint density function

$$f(x,y) = \begin{cases} \frac{1}{8}(6-x-y); 0 < x < 2, 2 < y < 4 \\ 0, \qquad otherwise. \end{cases}$$

Find (i) $P[X < 1 \cap Y < 3]$, (ii) $P[X + Y < 3]$, (iii) $P[X < 1 | Y < 3]$.

(9+9)

6.

a) In a normal distribution, 31 % of the items are under 45 and 8 % are over 64. Find the mean and standard deviation of the distribution.

b) In a partially destroyed laboratory record of an analysis of a correlation data (X,Y), the following results only are legible: Variance of $X = 9$. Regression equations: $8x - 10y + 66 = 0,\ 40x - 18y = 214$. What were (i) the mean values of X and Y (ii) the standard deviation of Y and the co-efficient of correlation between X and Y.

c) If X is the number scored in a throw of a fair die, show that the chebyshev's inequality gives $P[|X - \mu| > 2.5] < 0.47$, while the actual probability is zero.

(6+6+6)

7.

a) Let the sample values 2,3,5,6,8,9,11 and 12 are drawn from a uniform population, and let the unknown parameter of minimum value be 'a' and the parameter of maximum value be 'b' in the population. Estimate the range (a , b) of this population using the method of moments.

b) A die is thrown 276 times and the results of these throws are given below:

No. appeared on the die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test whether the die is biased or not.

c) To test the effectiveness of inoculation against cholera, the following table was obtained:

	Attacked	Not Attacked	Total
Inoculated	30	160	190
Not Inoculated	140	460	600
Total	170	620	790

(The figures represent the number of persons.)

Use Chi^2-test (Chi square - test) to defend or refute the statement that the inoculation prevents attack from cholera.

(6+6+6)