C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) There are 3 true coins and 1 false with *'heads'* on both sides. Each coin is equally likely. A coin is chosen at random and tossed 4 times. If *'head'* occurs all the 4 times, find the probability that the false coin has been chosen.

b) Is it possible to find Fourier transform of $\sin\left(\frac{1}{x}\right)$? Justify.

c) Formulate Kuhn–Tucker conditions for the following non linear programming problem

Minimize $x_1^2 - 4x_1 + x_2^2 - 6x_2$

Subject to the constraints

$2x_1 - x_2 \geq 2;$

$x_1 + x_2 \leq 3$

d) In a showroom, if customers arrival follows Poisson distribution with average arrival rate of λ, then prove that customer inter arrival time follows exponential distribution with mean $1/\lambda$.

e) Let {X(t)} be a Wide Sense Stationary process such that its autocorrelation function is $4e^{-2|\tau|}$. Find $E[\{X(t+1)-X(t-1)\}^2]$.

f) Write dual of the following linear programming problem

Maximize $z = 3x_1 + 8x_2$

Subject to the constraints

$x_1 + x_2 \leq 5;\ x_1 + x_2 \geq 6;\ x_1 + 2x_2 \leq 4;\ x_1, x_2 \geq 0$. Discuss optimum value of objective of the dual.

g) Find inverse Laplace transform of the function $\dfrac{4e^{-(s\pi/2)}}{s^2 + 16}$.

(7x4)

2.

a) Two joint normal random variables X and Y are uncorrelated. Derive an expression for joint entropy of (X, Y). Hence find the joint entropy of (X, Y), if variances of X and Y are 4 and 9 respectively.

b) A random variable *X* has the following probability distribution

x	0	1	2	3	4	5	6	7	otherwise
p(*x*)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2$+K	0

Find

i) the value of K

ii) P(1.5< *X*<4.5 / X>2)

iii) smallest value of λ such that $P(X \leq \lambda) > 1/2$.

(9+9)

3.

a) Solve the following integer programming problem

Maximize $z = 5x_1 + 4x_2$

Subject to conditions

$x_1 + x_2 \le 5;$

$10x_1 + 6x_2 \le 45;$

x_1 and x_2 are non negative integer;

by using the branch and bound algorithm. Take x_1 as initial branching variable.

b) Given an average arrival rate of 20 per hour. During steady state, Is it better for a customer to get service at a single channel with mean service rate of 22 customers per hour or at one of two channels in parallel with mean service rate of 11 customers per hour for each of the two channels. Assume both queues to be of Poisson type.

(9+9)

4.

a) Let {X(t)} be a Poisson process such that E[X(3)]=2. Find

i) Mean and variance of X(9).

ii) P(X(2)≤3)

iii) P(X(4)≤5 / X(2) =3)

iv) E[X(2)X(4)]

b) Find the solution of the initial value problem

$y'' + ty' - 2y = 6 - t,\ y(0) = 0,\ y'(0) = 1$

by using the Laplace transform, given that Laplace transform of $y(t)$ exists.

(9+9)

5.

a) Find the Fourier series expansion of the function

$$f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ 0, & 0 \le x < \pi \end{cases}.$$

Hence prove that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \Lambda = \frac{\pi^2}{8}.$$

b) Using Simplex method, solve the following linear programming problem

Maximize $z = 2x_1 - x_2 + 2x_3$

Subject to conditions

$2x_1 + x_2 \le 10;$

$x_1 + 2x_2 - 2x_3 \le 20;$

$x_2 + 2x_3 \le 5;$

$x_1, x_2, x_3 \ge 0;;$

(9+9)

6.

a) Define a symmetric random walk. Derive an expression for its autocorrelation function.

b) The joint probability density function of a two-dimensional random variable (X,Y) is given by

$$f(x,y) = \begin{cases} k(6-x-y); & 0<x<2, 2<y<4 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) the valure of *k*

ii) P(X<1, Y<3)

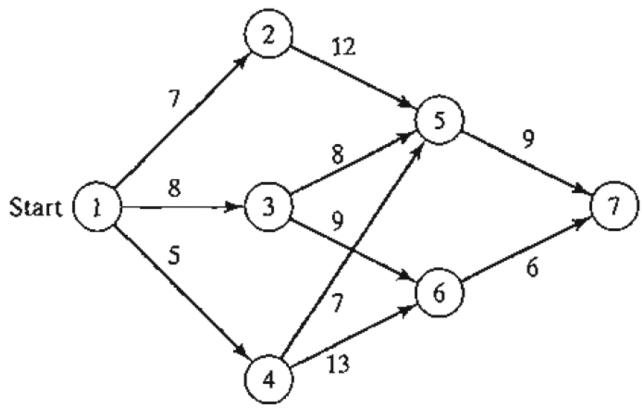
iii) P(X+Y<3)

iv) P(X<1 / Y<3).

(9+9)

7.

a) By using the backward recursion, find the shortest distance and shortest distance route from node 1 to node 7.



b) The local one-person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair-cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour. If time for hair-cut is exponentially distributed, find

i) percentage of idle time of the barber

ii) fraction of the potential customers turned away

iii) expected number of customers waiting for a hair-cut

iv) expected time a customer can spend in .the barber shop.

(9+9)