No. of Printed Pages : 4

Sl. No.

B0.1-R5 : BASIC MATHEMATICS

DURATION : 03 Hours **MAXIMUM MARKS : 100**

Roll No. : | | | | | | |

Answer Sheet No. : | | | | | | |

Name of Candidate : ____________________ ; **Signature of Candidate :** ____________________

INSTRUCTIONS FOR CANDIDATES :

- Carefully read the instructions given on Question Paper, Answer Sheet.
- Question Paper is in English language. Candidate has to answer in English Language only.
- **Question paper contains Seven questions. The Question No. 1 is compulsory. Attempt any FOUR Questions from Question No. 2 to 7.**
- **Parts of the same question should be answered together and in the same sequence.**
- **Questions are** to be answered in the **ANSWER SHEET** only, supplied with the Question Paper.
- **Candidate cannot leave the examination hall/ room without signing on the attendance sheet and handing over his/her Answer Sheet to the Invigilator. Failing in doing so, will amount to disqualification of Candidate in this Module/Paper.**
- After receiving the instruction to open the booklet and before answering the questions, the candidate should ensure that the Question Booklet is complete in all respects.

DO NOT OPEN THE QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.

1. (a) Using the Ratio test, test the convergence of the series whose n^{th} term is $\frac{2^n}{n^3}$.

(b) Evaluate $\int_0^1 x^4 \left(1-x^2\right)^{\frac{3}{2}} dx$.

(c) Find the asymptotes of the family of curves $x^3+3x^2y-4y^3-x+y+3=0$.

(d) Use Cauchy's Mean Value theorem to evaluate $\lim_{x\to 1}\left[\frac{\cos\frac{\pi x}{2}}{\log\left(\frac{1}{x}\right)}\right]$.

(e) Evaluate $\int_0^{\pi} \frac{\sqrt{(1-\cos x)}}{1+\cos x}\sin^2 x dx$.

(f) Find the Taylor series for cos $2x$ at $x=0$.

(g) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ w. r. to x. **(7x4)**

2. (a) Solve the following system of linear equations using the Gauss elimination method :

$$2x+y-z=5$$
$$x-3y+2z=-1$$
$$3x+2y-2z=6$$

(b) If $x=e^{-t^2}$ and $y=\tan^{-1}(2t+1)$, find $\frac{dy}{dx}$.

(c) Using Integral test, discuss the convergence of the following series : $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$

(9+6+3)

3. (a) If $f(x)=x\exp\left[-\left(\frac{1}{|x|}+\frac{1}{x}\right)\right]$; $x \neq 0$ and $f(x)=0$; $x=0$ then test

(i) whether $f(x)$ is continuous at $x=0$, and

(ii) $f(x)$ is differentiable at $x=0$.

(b) Find the vector projection of a force $\vec{F} = 5\hat{i} + 2\hat{j}$ onto $\vec{v} = \hat{i} - 3\hat{j}$ and the scalar component of $\vec{F}$ in the direction of $\vec{v}$. **(9+9)**

4. (a) Solve the following system of linear equations using the Cramer's rule :

$$3x-y+2z=4$$
$$2x+2y-3z=7$$
$$x+4y-z=5$$

(b) Find the volume of the solid generated by revolving the region between the parabola $x=y^2+1$ and the line $x=3$ about the line $x=3$. **(9+9)**

5. (a) Find the area enclosed by the curve $a^2x^2 = y^3\ (2a - y)$.

(b) Find the distance from the point S(1, 1, 5) to the line L : $x = 1 + t,\ y = 3 - t,\ z = 2t$.

(9+9)

6. (a) Find the eigenvalues and eigenvectors of the following matrix :

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

(b) Evaluate $I_n = \int_0^a \left(a^2 - x^2\right)^n dx$ where n is a positive integer. Hence show that

$$I_n = \frac{2n}{2n + 1} a^2 I_{n-1}$$

(12+6)

7. (a) Show that the function f defined by $f(x) = x^p(1 - x)^q\ \forall x \in R$ has a maximum value for $x = \frac{p}{p + q}$ for all p and q, where p and q are positive integers.

(b) Find the area inside the smaller loop of $r = 2\cos\theta + 1$.

(9+9)

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SPACE FOR ROUGH WORK