No. of Printed Pages : 4

Sl. No.

C0-R4.B1 : ELEMENTS OF MATHEMATICAL SCIENCES

DURATION : 03 Hours **MAXIMUM MARKS : 100**

Roll No. : | | | | | | |

Answer Sheet No. : | | | | | | |

Name of Candidate : _____ **; Signature of Candidate :** _____

INSTRUCTIONS FOR CANDIDATES :

- Carefully read the instructions given on Question Paper, Answer Sheet.
- Question Paper is in English language. Candidate has to answer in English Language only.
- **Question paper contains Seven questions. The Question No. 1 is compulsory. Attempt any FOUR Questions from Question No. 2 to 7.**
- **Parts of the same question should be answered together and in the same sequence.**
- **Questions are** to be answered in the **ANSWER SHEET** only, supplied with the Question Paper.
- **Candidate cannot leave the examination hall/ room without signing on the attendance sheet and handing over his/her Answer Sheet to the Invigilator. Failing in doing so, will amount to disqualification of Candidate in this Module/Paper.**
- After receiving the instruction to open the booklet and before answering the questions, the candidate should ensure that the Question Booklet is complete in all respects.

DO NOT OPEN THE QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.

1. (a) Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$. Find the rank of $P+Q$.

(b) Given the matrix $A = \begin{bmatrix} a & b \\ 2 & \frac{7-11a}{3} \end{bmatrix}$, Trace of $A=5$, and the matrix columns are linearly dependent or full rank. Find the values of a and b.

(c) Find the value of $f(x)$ using the relation $f\left(\frac{x+2}{x-2}\right) = \frac{x^2+4x+4}{8x}$.

(d) Evaluate the expression $\lim_{x\to\infty}[e^x + x]^{1/x}$.

(e) Find the intersection of the point when the straight-line $y=2x+1$ interact with circle $(x-2)^2+(y-3)^2=4$

(f) Using the limits of the sequence find the value of N and prove the relation $\frac{n}{n+1} = 1+10^{-5}$ for n>N

(g) Suppose you have two bags of marbles. Bag A contains 4 red marbles and 3 green marbles, while Bag B contains 2 red marbles and 5 green marbles. You randomly choose one bag and then randomly draw a marble from it. Given that you draw a green marble, what is the probability that you chose Bag A ? **(7x4)**

2. (a) A company claims that the distribution of its product sales across different regions follows a uniform distribution. To test this claim, a sample of 100 sales records is taken, and the number of sales in each region is recorded. Perform a chi-square goodness-of-fit test at a 5% significance level to determine whether the observed distribution differs significantly from the claimed uniform distribution.

(b) A call center receives an average of 10 calls per hour. What is the probability that they will receive exactly 5 calls in a given hour using Poisson distribution ? **(9+9)**

3. (a) Find the Maclaurin series expansion of the function $f(x)=e^x\sin(x)$ and determine its radius of convergence.

(b) Find two quadratic equations with integer coefficients that have the roots 2 and -3.

(c) Evaluate $\int \frac{\sin 2x}{1+\sin x}$

(9+4+5)

4. (a) Solve the following system of equations using Cramer's rule :

$x + y + z = 6$

$2x + 3y - z = 5$

$6x - 2y - 3z = -7$

(b) Suppose we have the following data representing the number of hours studied (independent variable) and the corresponding test scores (dependent variable) for a group of students :

Hours Studied (x)	**Test Score (y)**
2	65
3	75
4	82
5	88
6	92

Perform a linear regression analysis to find the equation of the regression line and use it to predict the test score for a student who studies for 7 hours. **(10+8)**

5. (a) In a survey of two students it is found that A speaks truth in 75% cases and B in 80% cases. In what percent of cases are they likely to contradict each other in narrating the same event ?

(b) If the letters of the word 'ASSASSIN' are written down at random in a row. What is the probability that in the written word 2 'A' occur together ?

(c) Determine the partial fraction decomposition of the following expression :

$$\frac{4x^3 + 16x + 7}{(x^2 + 4)^2}$$

(6+6+6)

6. (a) Find the eigenvalues and eigen vectors of the following 3×3 matrix :

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

(b) Suppose we have three variables X, Y, and Z, and their corresponding values are given by the following table :

	X	**Y**	**Z**
X	2	3	4
Y	4	5	6
Z	6	7	8

Calculate the covariance matrix for the variables X, Y, and Z. **(10+8)**

7. (a) Find all the asymptotes of the curve :

$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$

(b) Find the value of the P for which the vector $A = 2i + Pj + k$ and $B = 4i - 2j - 2k$ are perpendicular to each other. **(12+6)**

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SPACE FOR ROUGH WORK