No. of Printed Pages : 6

Sl. No.

C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

DURATION : 03 Hours **MAXIMUM MARKS : 100**

Roll No. : | | | | | | |

Answer Sheet No. : | | | | | | |

Name of Candidate : ________________ ; **Signature of Candidate :** ________________

INSTRUCTIONS FOR CANDIDATES :

- Carefully read the instructions given on Question Paper, Answer Sheet.
- Question Paper is in English language. Candidate has to answer in English Language only.
- **Question paper contains Seven questions. The Question No. 1 is compulsory. Attempt any FOUR Questions from Question No. 2 to 7.**
- **Parts of the same question should be answered together and in the same sequence.**
- **Questions are** to be answered in the **ANSWER SHEET** only, supplied with the Question Paper.
- **Candidate cannot leave the examination hall/ room without signing on the attendance sheet and handing over his/her Answer Sheet to the Invigilator. Failing in doing so, will amount to disqualification of Candidate in this Module/Paper.**
- After receiving the instruction to open the booklet and before answering the questions, the candidate should ensure that the Question Booklet is complete in all respects.

DO NOT OPEN THE QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.

1. (a) Find the Fourier Transform of the signal $e^{-|a|t} u(t)$.

(b) It is estimated that 50% of emails are spam. Some software has been applied to filter these spam emails before they reach your inbox. A certain software brand claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email ?

(c) Let's consider a system with three possible states, A, B, and C, with probabilities P(A) = 0.4, P(B) = 0.3, and P(C) = 0.3. Find the entropy H(X) of a discrete random variable X in such a system.

(d) Define Brownian motion. List out some of the properties of Brownian motion.

(e) Compute the derivative of the function $y = \arcsin(2x+1)$.

(f) Evaluate $\int \frac{\sec^4(2t)}{\tan^9(2t)} dt$

(g) Machine A has an interactive timesharing system of 20 active terminals which can be studied by the machine repair model. The average CPU service time, including swapping, is 2 seconds, while the mean think time is 20 seconds. Find p_0, ρ, λ, and the average response time W. (λ is the average throughput in interactions per second). **(7x4)**

2. (a) Find the Fourier Transform of the signal given below and draw the magnitude and phase response :

(i) $x(t) = \begin{cases} A; & |t| < T_0 \\ 0; & |t| > T_0 \end{cases}$

(ii) $x(t) = e^{-at} u(t)$

(b) Consider a particle undergoing one-dimensional Brownian motion starting at position $x = 0$ at time $t = 0$. The particle has a diffusion coefficient $D = 1$. What is the probability that the particle is between positions $x = -1$ and $x = 1$ after $t = 1$ unit of time ? **(12+6)**

3. Let the random variable X be five possible symbols {$\alpha, \beta, \gamma, \delta, \epsilon$}. Consider two probability distributions p(x) and q(x) over these symbols, and two possible coding schemes C1(x) and C2(x) for this random variable :

Symbol	p(x)	q(x)	C1(x)	C2(x)
α	1/2	1/2	0	0
β	1/4	1/8	10	100
γ	1/8	1/8	110	101
δ	1/16	1/8	1110	110
ϵ	1/16	1/8	1111	111

(a) Calculate H(p), H(q), and relative entropies (Kullback-Leibler distances) $D(p||q)$ and $D(q||p)$.

(b) Show that the average codeword length of C1 under p is equal to H(p), and thus C1 is optimal for p. Show that C2 is optimal for q.

(c) Now assume that we use code C2 when the distribution is p. What is the average length of the codewords? By how much does it exceed the entropy H(p)? Relate your answer to D(p||q).

(d) If we use code C1 when the distribution is q, by how much does the average codeword length exceed H(q)? Relate your answer to D(q||p). **(4.5x4)**

4. (a) A two-state Markov process may emit '0' in State 0 or emit '1' in State 1, each with probability α, and return to the same state; or with probability $(1-\alpha)$ it emits the other symbol and switches to the other state.

(i) Draw the two-state Markov process diagram using the above information.

(ii) What are the state occupancy probabilities for $0 < \alpha < 1$?

(iii) What are the entropy of State 0, the entropy of State 1, and the overall entropy of this source ? Express your answers in terms of α.

(iv) For what value(s) of α do both forms of predictability disappear ? What then is the entropy of this source, in bits per emitted bit ?

(b) Consider the joint distribution of two random variables X and Y.

Let $p(x, y) = \Pr[X = x \wedge Y = y]$ and $p(x)$ and $p(y)$ be $\Pr[X = x]$ and $\Pr[Y = y]$, respectively. Prove that, $I(X; Y) = D(p(x, y) || p(x)p(y))$. Note that $p(x)p(y)$ is the probability that x and y are chosen from the product distribution of the marginal distributions for X and Y. **(12+6)**

5. (a) In a survey of two students it is found that A speaks truth in 75% cases and B in 80%cases. In what percent of cases are they likely to contradict each other in narrating the same event ?

(b) If the letters of the word 'ASSASSIN' are written down at random in a row. What is the probability that in the written word 2 'A' occur together ?

(c) The position of an object at any time t is given by $s(t) = 3t^4 - 40t^3 + 126t^2 - 9$.

(i) Determine the velocity of the object at any time t.

(ii) Does the object ever stop changing ?

(iii) When is the object moving to the right and when is the object moving to the left ? **(6+6+6)**

6. (a) Define the Integer Programming with Branch-and-Bound Technique. Write down the steps to solve the problem.

(b) Given a function, Max $Z = -x_1 + 4x_2$, subject to the given conditions:

$-10x_1 + 20x_2 \leqslant 22$

$5x_1 + 10x_2 \leqslant 49$

$x_1 \leqslant 5$

$x_i \geqslant 0$, x_i's are integers. Find the solution using the Integer Branch and Bound technique. **(6+12)**

7. For a small batch computing system, the processing time per job is exponentially distributed with an average time of 3 minutes. Jobs arrive randomly at an average rate of one job every 4 minutes and are processed on a first-come-first-served basis. Give the proper solution of the manager related to the installation.

(a) What is the probability that an arriving job will require more than 20 minutes to be processed (the job turn-around time exceeds 20 minutes) ?

(b) A queue of jobs waiting to be processed will form, occasionally. What is the average number of jobs waiting in this queue ?

(c) It is decided that when the workload increases to the level such that the average time in the system reaches 30 minutes, the computer system capacity will be increased. What is the average arrival rate of jobs per hour at which this will occur? What is the percentage increase over the present job load ? What is the average number of jobs in the system at this time ?

(d) Suppose the criterion for upgrading the computer capacity is that not more than 10% of all jobs have a time in the system (turn-around time) exceeding 40 minutes. At the arrival rate at which this criterion is reached, what is the average number of jobs waiting to be processed ?

(3+5+5+5)

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