

BO.1-R5 : BASIC MATHEMATICS

DURATION : 03 Hours

MAXIMUM MARKS : 100

Roll No. :

Answer Sheet No. :

Name of Candidate : _____ ; **Signature of Candidate :** _____

INSTRUCTIONS FOR CANDIDATES :

- Carefully read the instructions given on Question Paper, Answer Sheet.
- Question Paper is in English language. Candidate has to answer in English Language only.
- Question paper contains Seven questions. The Question No. 1 is compulsory. Attempt any FOUR Questions from Question No. 2 to 7.
- Parts of the same question should be answered together and in the same sequence.
- Questions are to be answered in the ANSWER SHEET only, supplied with the Question Paper.
- Candidate cannot leave the examination hall/ room without signing on the attendance sheet and handing over his/her Answer Sheet to the Invigilator. Failing in doing so, will amount to disqualification of Candidate in this Module/Paper.
- After receiving the instruction to open the booklet and before answering the questions, the candidate should ensure that the Question Booklet is complete in all respects.

DO NOT OPEN THE QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.

1. (a) Test the convergence of the alternating series : $\sum_{n=1}^{\infty} \frac{2n^3+5}{4n^5+1}$.

(b) If $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Find $|A|$. Also find the rank of A.

(c) Find the centre and radius of the circle $x^2 + y^2 + 3x - 5y + \frac{9}{4} = 0$.

(d) Find the angle between the vectors $2\hat{i} + 3\hat{j} - 3\hat{k}$ and $3\hat{i} + 2\hat{j} + 4\hat{k}$.

(e) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$.

(f) If $x = a(\theta + \sin\theta)$; $y = a(1 - \cos\theta)$. Find $\frac{dy}{dx}$.

(g) Evaluate $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$.

(7x4)

2. (a) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1.3.5....(2n-1)}{2.4.6....2n}$.

(b) Using Integral test, discuss the converge of the series $\sum_{n=1}^{\infty} \frac{8 \tan^{-1} n}{1+n^2}$.

(c) Find the sum of all three-digit numbers which are divisible by 7.

(6+6+6)

3. (a) Find the inverse of the matrix $\begin{bmatrix} -1 & 2 & 2 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$.

(b) Solve the system of simultaneous linear equations by Cramer's rule :

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

(c) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

(6+6+6)

4. (a) Find the joint equation of a pair of lines through the origin and perpendicular to the pair of lines $2x^2 + 3xy + y^2 = 0$.
 (b) Find the axis, foci, eccentricity and length of latus rectum of the conic $2x^2 + 3y^2 = 1$.
 (c) Find the equation of the tangent and normal to the circle $x^2 + y^2 = 13$ at the point $(3, -2)$. (6+6+6)

5. (a) Find a unit vector perpendicular to the vectors $\vec{a} - \vec{b}$ and $\vec{b} + \vec{c}$, where $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = -\hat{i} + \hat{j} + \hat{k}$
 (b) Parametrize the line segment joining the points $P(-3, 2, -3)$ and $Q(1, -1, 4)$.
 (c) If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, find the value of $\frac{d^2y}{dx^2}$ for $x=a$.
 (d) Evaluate $\int_1^3 \frac{\ln x}{x} dx$. (5+4+5+4)

6. (a) If $\sin y = x \sin(a+y)$ then show that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.
 (b) If $x+y=4$, then find the maximum value of xy^2 .
 (c) Find all the asymptotes of the family of curves $x^2y + xy^2 + xy + y^2 + 3x = 0$. (6+6+6)

7. (a) Integrate the function $\tan^{-1} x$ with respect to x .
 (b) Evaluate $\int_0^{\pi/4} \log(1 + \tan x) dx$.
 (c) Find the area between the curve $y^2 = x$ and the line $y=x$. (6+6+6)

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SPACE FOR ROUGH WORK