No. of Printed Pages : 4

Sl. No.

B1.2-R5 : DISCRETE STRUCTURES

DURATION : 03 Hours **MAXIMUM MARKS : 100**

Roll No. : | | | | | | |

Answer Sheet No. : | | | | | | |

Name of Candidate : _____ **; Signature of Candidate :** _____

INSTRUCTIONS FOR CANDIDATES :

- Carefully read the instructions given on Question Paper, Answer Sheet.
- Question Paper is in English language. Candidate has to answer in English Language only.
- **Question paper contains Seven questions. The Question No. 1 is compulsory. Attempt any FOUR Questions from Question No. 2 to 7.**
- **Parts of the same question should be answered together and in the same sequence.**
- **Questions are** to be answered in the **ANSWER SHEET** only, supplied with the Question Paper.
- **Candidate cannot leave the examination hall/ room without signing on the attendance sheet and handing over his/her Answer Sheet to the Invigilator. Failing in doing so, will amount to disqualification of Candidate in this Module/Paper.**
- After receiving the instruction to open the booklet and before answering the questions, the candidate should ensure that the Question Booklet is complete in all respects.

DO NOT OPEN THE QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.

1. (a) Let R be the equivalence relation in the set Z of integers given by R = {(a, b) : 3 divides a − b}. Write the equivalence class [0].

(b) Consider that in a library of 500 students, every student reads 4 newspapers, and every newspaper is read by 50 students. What is the number of newspapers ?

(c) Show that the relation R = {(a, a), (a, b), (b, a), (b, b), (c, c)} on A = {a, b, c} is an equivalence relation.

(d) Explain the following terms with examples :
(i) Conjunction (AND) and (ii) Tautology

(e) Define an algorithm. What is the need for the analysis of algorithms ?

(f) Assume that a company has 50 employees, and each employee has a company ID number between 1 and 45. Show that at least two employees must have the same ID.

(g) Write the types of Binary Trees with examples. **(7x4)**

2. (a) What do you understand by graph theory ? Explain the basic components of a graph.

(b) What are the basic differences between a graph and a tree ? Explain with an example.

(c) Explain various types of binary tree traversal methods. **(6+6+6)**

3. (a) Consider a weighted undirected graph G = (V, E) with the following vertices and edges, where weights represent distances in kilometers :
V = {A, B, C, D, E}
E = {(A, B, 4), (A, C, 2), (B, C, 1), (B, D, 5), (C, D, 8), (C, E, 10), (D, E, 2)}
(i) Draw the graph and represent it using an adjacency matrix and an adjacency list.
(ii) Identify one simple path from A to E and one cycle in the graph. What are their lengths (sum of edge weights) ?
(iii) Using a brute force approach, find the shortest possible route that starts at A, visits every vertex exactly once, and returns to A. Calculate the total distance.

(b) What is a cut set in a graph ? Explain the real life applications of cut sets. **(12+6)**

4. (a) Why do we use asymptotic notation in the study of algorithm ? Describe commonly used asymptotic notations and give their significance.

(b) Given an array A [ ] = {15, 21, 27, 36, 47, 51, 63, 70}, apply both Merge Sort and Insertion Sort to sort the given array in ascending order. Determine which algorithm requires the fewest comparisons and explain why ? **(9+9)**

5. (a) Explain the divide and conquer approach. Also, list out various advantages and disadvantages of the divide and conquer approach.

(b) Find the Longest Common Subsequence (LCS) for the two sequences given below using Dynamic programming :

X = "ACDBE"

Y = "ABCDE"

(8+10)

6. (a) A divide and conquer algorithm P splits the original problem (which is stored as a matrix) in two sub-problems of equal size of n/2, and then the solutions of the sub-problems is to be combined to get the solution for the original problem. What's the running time of P ? Show the steps.

(b) Devise a solution of Matrix Chain Multiplication (A, B, C, D) using dynamic programming for the dimensions' matrix [2, 5, 3, 7, 4].

(10+8)

7. (a) In a group of 50 students, 30 take Math, 25 take Physics, and 15 take both. How many students take at least one of these subjects, and how many take neither ?

(b) A license plate consists of 2 letters followed by 4 digits. Letters are chosen from A-Z (26 options), and digits are chosen from 0-9 (10 options). However, the first letter cannot be O (to avoid confusion with zero), and repetition is allowed. How many possible license plates can be created ?

(c) Consider the following three mathematical functions based on their asymptotic growth rates as the input n approaches infinity. The goal is to arrange them in decreasing order of their big-O complexity, meaning the function with the fastest growth rate should be listed first, and the function with the slowest growth rate should be listed last :

$f1(n) = n^{\sqrt{n}}$, $f2(n) = 2^n$, and $f3(n) = n^{(10)} * 2^{(n/2)}$

(6+6+6)

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SPACE FOR ROUGH WORK