Sl. No.

C0-R4.B1 : ELEMENTS OF MATHEMATICAL SCIENCES

DURATION : 03 Hours   **MAXIMUM MARKS : 100**

Roll No. :   **Answer Sheet No. :**

Name of Candidate : _____ ; **Signature of Candidate :** _____

INSTRUCTIONS FOR CANDIDATES :

• Carefully read the instructions given on Question Paper, Answer Sheet.

• Question Paper is in English language. Candidate has to answer in English Language only.

• **Question paper contains Seven questions. The Question No. 1 is compulsory. Attempt any FOUR Questions from Question No. 2 to 7.**

• **Parts of the same question should be answered together and in the same sequence.**

• **Questions are** to be answered in the **ANSWER SHEET** only, supplied with the Question Paper.

• **Candidate cannot leave the examination hall/ room without signing on the attendance sheet and handing over his/her Answer Sheet to the Invigilator. Failing in doing so, will amount to disqualification of Candidate in this Module/Paper.**

• After receiving the instruction to open the booklet and before answering the questions, the candidate should ensure that the Question Booklet is complete in all respects.

DO NOT OPEN THE QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.

1. (a) Find the square root of the complex number (i) $A = 1 + i$ and (ii) $B = 1 - i$

(b) Find the partial fraction of the expression :

$$\frac{2x^3 - 4x^2 + 3x + 1}{(x-1)^2}$$

(c) Find the limit of the following function as x approaches infinity : $\lim\limits_{x \to \infty} \dfrac{e^x}{x^2}$.

(d) Consider the following 3×3 matrix A : $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

Determine whether the rank of matrix A is 1 or not.

(e) Find the intersection of the point when the straight line $y = 2x + 1$ interact with circle $(x-3)^2 + (y-2)^2 = 4$

(f) In a game, two fair six-sided dice are rolled. What is the probability that the product of the numbers rolled is a perfect square?

(g) Evaluate the following integral and find the value : $\lim\limits_{n \to \infty} \int_0^1 \dfrac{x^n}{1+x} \, dx$ **(7x4)**

2. (a) A box contains "a" white and "b" black balls, of which "c" balls are drawn. Find the expected value of the number of white balls drawn ?

(b) In a certain factory turning out blades, there is a 0.2% probability of any blade to be defective. Blades are supplied in packets of 10. Using Poisson distribution, calculate the approximate number of packets containing no defects, one defective, two defective, three defective blades, respectively, in a consignment of 20,000 packets. $(e^{-0.02} = 0.9802)$

(c) Using the Binomial distribution, answer the following :

(i) Check the statement as truth/fallacy, for the given statement that the mean of the distribution is 15 and its standard deviation is 5.

(ii) If the mean and the variance of the distribution are 9 and 6, respectively, find the distribution.

(iii) If the sum of the mean and variance of the distribution for 5 trials is 1.8, find the distribution.

(iv) The mean of the distribution is 20, and the standard deviation is 4. Calculate the values of n, p, and q. **(4+6+8)**

3. (a) In an experiment, the following results relating to immunization from T.B. of animals is obtained :

	Died or affected	Unaffected	Total
Inoculated	12	24	36
Not inoculated	16	8	24
Total	28	32	60

Test the effect of inoculation on the control of T.B. using Chi-square test.

(b) Find the first 4 terms in the Taylor series for the given function :
$F(x) = (x-1)e^x$ near $x = 1$ **(9+9)**

4. (a) Compute the first 4 terms in the Maclaurin series for the function $\dfrac{x+1}{x^2-5x+6}$ by first finding the partial fraction decomposition of the function.

(b) Determine the number of solutions for the given system of equations using Cramer's rule :

$$x - 2y + 3z = 17$$
$$2x + y + 2z = 6$$
$$2x - 4y + 6z = 34$$

(c) Explain the Fundamental Theorem of Calculus. Show the explanation with an example.

(7+7+4)

5. (a) Consider the parameterized curve given by the equations :
$X(t) = 3\cos(t)$, $Y(t) = 2\sin(t)$, where t is a parameter in the interval $[0, 2\pi]$

1. Find the Cartesian equation of the curve.

2. Determine the slope of the tangent line to the curve at the point where

$$t = \frac{\pi}{4}$$

3. Find the arc length of the curve over the interval $[0, 2\pi]$.

(b) Find the sum of the infinite series $\dfrac{1}{1} - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \dfrac{1}{5} - \ldots\ldots = ?$

(12+6)

6. (a) Using the data given in a sample table for a random experiment, answer the following :

Variable	Experiment 1	Experiment 2	Experiment 3	Experiment 4
X_1	4	8	13	7
X_2	11	4	5	14

(i) Find the Covariance Matrix of the sample.

(ii) Find the Eigen values and corresponding Eigen vectors for the given data.

(b) Find the derivative of the function with respect to x when y is given as :

$$y = \frac{(x + 3)^3 . \tan(x)}{(3x + 7)^2}$$

(12+6)

7. (a) Find all the asymptotes of the curve :
$(x + y)^2 (x + 2y + 2) = x + 9y - 2$

(b) Given two vectors $u = 2i + 5j + 9k$ and $v = 7i + j + 2k$, find the value of the constant P when :

(i) Two vectors are perpendicular and follow the condition $u.v = P.\|v\|^2$

(ii) Two vectors are collinear.

(9+9)

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SPACE FOR ROUGH WORK