No. of Printed Pages : **7**

Sl. No.

C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

DURATION : 03 Hours **MAXIMUM MARKS : 100**

Roll No. : | | | | | | |

Answer Sheet No. : | | | | | | |

Name of Candidate : ______________________ ; **Signature of Candidate :** ____________

INSTRUCTIONS FOR CANDIDATES :

- Carefully read the instructions given on Question Paper, Answer Sheet.
- Question Paper is in English language. Candidate has to answer in English Language only.
- **Question paper contains Seven questions. The Question No. 1 is compulsory. Attempt any FOUR Questions from Question No. 2 to 7.**
- **Parts of the same question should be answered together and in the same sequence.**
- **Questions are** to be answered in the **ANSWER SHEET** only, supplied with the Question Paper.
- **Candidate cannot leave the examination hall/ room without signing on the attendance sheet and handing over his/her Answer Sheet to the Invigilator. Failing in doing so, will amount to disqualification of Candidate in this Module/Paper.**
- After receiving the instruction to open the booklet and before answering the questions, the candidate should ensure that the Question Booklet is complete in all respects.

DO NOT OPEN THE QUESTION BOOKLET UNTIL YOU ARE TOLD TO DO SO.

1. (a) The counter of a bank performs the transactions with a mean time of 2 minutes. The customers arrive at a mean rate of 20 customers/hour. Assuming arrivals adhere to a Poisson process and service times are exponentially distributed, determine :

(i) Percentage of the time the bank teller is idle

(ii) Mean waiting time of the customers

(iii) Percentage of customers that wait in a queue

(b) Suppose that women who live beyond the age of 80 outnumber men in the same age group by three to one. How much information, in bits, is gained by learning that a person who lives beyond 80 is male ?

(c) Find the inverse Laplace Transform of $F(p) = \dfrac{2p}{(p^2 + 1)(p + 1)}$

(d) Suppose that the chance of rain tomorrow depends on previous weather conditions only through whether or not it is raining today and not on past weather conditions. Suppose also that if it rains today, then it will rain tomorrow with probability α; and if it does not rain today, then it will rain tomorrow with probability β. Design the two state Markov chain with transition probabilities.

(e) Prove that the information measure is additive: that the information gained from observing the combination of N independent events, whose probabilities are p_i for $i = 1....N$, is the sum of the information gained from observing each one of these events separately and in any order.

(f) Solve, min $J = x_1^2 + x_2^2 + x_3^2 + x_4^2$

(g) Find the Fourier transform of $x(t) = e^{-|t|}\cos(2t)$

(7x4)

2. (a) Consider the function $f(t) = \begin{cases} t, & t \in [0, 1); \\ 1, & t \in [1, 2). \end{cases}$ Find its Fourier series and its sum.

(b) Use the Laplace transform to solve the given problem

$$\ddot{y} + y = \begin{cases} 1, & t \in [0, \pi) \\ 0, & \text{elsewhere} \end{cases} \quad y(0_+) = 2, \;\; \dot{y}(0_+) = 1$$

(9+9)

3. (a) A conservative design team, call it C, and an innovative design team, call it N, are asked to separately design a new product within a month. From previous experience we know that :

(i) The probability that team C is successful is 2/3.

(ii) The probability that team N is successful is 1/2.

(iii) The probability that at least one team is successful is 3/4.

If both teams are successful, the design of team N is adopted.
Assuming that exactly one successful design is produced, what is the probability that it was designed by team N ?

(b) The citizens of a locality withdraw money from an ATM according to following probability function (X) :

Amount, x in Rs.	50	100	200
P(X=x)	0.3	0.5	0.2

The number of customers per day has the distribution N ~ Poisson(λ).

Let $T_N = X_1 + X_2 + \ldots + X_N$ be the total amount of money withdrawn in a day, where each X_i has the probability function above, and $X_1, X_2, \ldots$ are independent of each other and of N. T_N is a randomly stopped sum, stopped by the random number of N customers.

(i) Show that E(X) = 105, and Var(X) = 2725.

(ii) Find $E(T_N)$ and $Var(T_N)$: the mean and variance of the amount of money withdrawn each day.

(9+9)

4. (a) The owner of a machine shop is planning to expand his business by purchasing some new machines - presses and lathes. The owner has estimated that each press purchase will increase profit by Rs. 100 per day and each lathe will increase profit by Rs. 150 daily. The number of machines the owner can purchase is limited by the cost of the machines and the available floor space in the shop. The machine purchase price and space requirements are as follows.

Machine	Required Floor Space (ft^2)	Purchase Price
Press	15	Rs. 8,000
Lathe	30	Rs. 4,000

The owner has a budget of Rs. 40,000 for purchasing machines and 200 square feet of available floor space. Help the owner to know how many of each type of machine to purchase to maximize the daily increase in profit.

(b) A gram sabha must decide which recreation facilities to construct in its community. Four new recreation facilities have been proposed- a swimming pool, an archery center, an athletic field, and a gymnasium. The council wants to construct facilities that will maximize the expected daily usage by the residents of the community subject to land and cost limitations. The expected daily usage, cost, and land requirements for each facility are as follows:

Recreation Facility	Expected Usage (people/day)	Cost (Rs.)	Land Requirements (acres)
Swimming pool	300	35,000	4
Archery Center	90	10,000	2
Athletic field	400	25,000	7
Gymnasium	150	90,000	3

The community has a Rs. 120000 construction budget and 12 acres of land. As the swimming pool and archery center must be built on the same part of the land, however, only one of these two facilities can be constructed. Solve the problem to help the gram sabha, to know which of the recreation facilities to construct to maximize the expected daily usage.

(9+9)

5. (a) Let X, Y be jointly normal, independent random variables with pdf

$$f_{x,y}(x,y) = \frac{1}{2\pi\sigma_1\sigma_2} \exp\left\{-\frac{1}{2}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2 + \left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]\right\}$$

(i) Show that X and Y are independent random variables.

(ii) Find the marginal pdfs of X and Y.

(iii) Show that $E[X+Y] = \mu_1 + \mu_2$

(b) In a restaurant, customers are arriving at a rate of 100 per hour and take 30 seconds to be served. Answer the following questions-

(i) How much time do customers spend in the restaurant ?

(ii) How much time waiting in line ?

(iii) How many customers are there in the restaurant at any given time ?

(iv) What is the utilization factor ?

(9+9)

6. (a) A Markov chain has transition probability matrix as

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 \\ 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \end{bmatrix}$$

(i) Draw the transition diagram.

(ii) Find, if there is any absorbing state.

(iii) Which are the communicating classes ?

(iv) Find a stationary distribution.

(v) What are the periods of states ?

(vi) Find out inessential states, if there exists any ?

(vii) Which states are recurrent ?

(viii) Which states are transient ?

(ix) Which states are positive recurrent ?

(b) What assumptions differentiate M/M/1, M/G/1, and M/M/c queuing models ?

(9+9)

7. (a) There are no possible locations where facilities can be opened to provide some service to m customers. If facility i, i=1, . . . , n, is opened, a fixed cost f_i must be paid. The cost for serving customer j with facility i is c_{ij} for i=1, . . . , n and j=1, . . . , m. Every customer must be served from exactly one facility (but the same facility can serve more customers). Decide which facilities should be opened and assign each customer to a facility at the minimum total cost.

(b) Use Karush-Kuhn-Tucker (KKT) condition to -

Maximize $f(x,y) = xy,$

Subject to $x + y^2 \leq 2$; $x,y \geq 0$

(9+9)

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SPACE FOR ROUGH WORK